Chapter 1 - Introduction and Literature Survey

1.1 Introduction

As this thesis is concerned with the production and heating of a plasma with electromagnetic waves, this chapter will consist of a general description of plasma wave phenomena and a background discussion of the two types of waves of which some experimental and theoretical aspects are presented in the latter chapters.

Apart from those formed by beam injection techniques, the vast majority of plasmas being studied are formed by using high-power electric fields. The glow discharge set up between two electrodes with a large potential difference between them in a low pressure gas was the forerunner of the arc discharge. A common example of a continuous arc is the mercury plasma (Tonks and Langmuir (1929), while pulse techniques have been used to form inert gas plasmas (Jephcott and Stocker (1962)). Controlled thermonuclear devices such as the "Stellarator" in the U.S.A., "Tokomak" in the U.S.S.R., and "Zeta" in the U.K. are examples of plasmas forming the secondary of a transformer, power being supplied by a pulse of energy in the primary. Plasmas produced by the passage of a shock wave have been investigated by Wilcox (1960), Brennan (1963), and others for the so-called $\underline{j} \times \underline{B}$ mode, and gas-dynamic shocks by Zel'dovich Ya. B. (1966) for example.

'Quiescent' plasmas (those thought to have a minimal number of instabilities) have been formed with alkali earths of caesium and potassium (Wong, Motley, and D'Angelo (1963)) and with Penning type discharges.

The above mentioned plasmas have been formed using D.C. or quasi-D.C. fields. The rest of this chapter will be restricted to a discussion of plasmas produced by oscillating fields. The first observation of a high-frequency discharge was reported by M.C. Gutton (1924) using a cylindrical tube filled with air at pressures of 5 to 500 millitorr. Wood and Loomis (1927) and Wood (1929) report similar observations in O_2 and Ar. The oscillating field in these experiments was provided by a spark gap oscillator.

The development of radar and high-frequency communications transmitters, accelerated by World War II, provided sources of high power, high-frequency energy which were used for forming plasmas for research, and for industrial applications in the form of the plasma torch (Reed (1961)).

Theoretical considerations of the physical mechanism for production of radio-frequency plasmas have been given by Allis (1956), Francis (1960), and Finzie (1966), among others. The processes involved are quite complicated and parallel plate geometry (E mode) or an infinite solenoid (B mode) are generally the only types considered.

Distinct from the plasma production mechanisms are the specific heating processes. A review paper by Berger et al. (1958) discusses three types of heating mechanisms as well as the collisional heating which is the dominant mechanism in all the previously discussed plasmas in this chapter. Transit time heating occurs as particles traverse the heating section sufficiently fast that they suffer no collisions during a transit time and acquire energy from the applied field. Acoustic heating becomes important in the event of a small effective mean-free path and the oscillating field produces density variations which result in the propagation of sound waves in the plasma. The third type of heating discussed propagation of electromagnetic waves in the plasma, some time will be spent in examining the spectrum of oscillations which can be sustained by a plasma in a magnetic field.

There are a number of books which are completely devoted to the description of waves in a plasma, some of the more well-known being Stix (1962), Denisse and Delcroix (1963), Akhiezer (1967), Allis, Buchsbaum, and Bers (1963), and Ginsberg (1964). Significant sections describing oscillations are given in the books by Spitzer (1962) and Shafranov (1967).

The general description of waves given below is basically of the same format as Shafranov, where the relation between the square of the refractive index is determined as a function of the frequency. Although this means that the effect of changes in density and the positions of the critical frequencies are somewhat disguised, the general shape of the dispersion curves is easily seen. "Allis" diagrams used by Stix and Spitzer, and using the ratio of the plasma frequency to the wave frequency as the abscissa (Denisse and Delcroix) are considered by the author of this thesis as disguising the physical nature of the waves although, perhaps, being more mathematically elegant.

For this introductory discussion, an infinite plasma in a uniform magnetic field is assumed, and the hydrodynamic approximation is used to describe the dispersion of the waves. To simplify the nature of the plasma motion, the limiting case of a cold plasma is assumed where the ion and electron pressures are zero. Although this removes the ion and electron modes of oscillation parallel to the magnetic field, the general shape of these modes will be shown in fig (1.1).

By applying Maxwell's equation to the 'generalized Ohms law', neglecting pressure, density, and temperature gradients, and neglecting collisions, as for the moment we are only interested in the real part of the wave number and frequency, a dispersion relation can be obtained expressing the frequency as a function of the wave number, the parameters characterizing the plasma, and the uniform D.C. magnetic field permeating the plasma. For convenience, the dispersion relation is determined as a function of the square of the refractive index $N^2 = \left(\frac{kc}{\omega}\right)^2$ (where $k = \frac{2\pi}{\lambda}$ is the wave number). The c.g.s. electrostatic system of units is used throughout this thesis to simplify some of the algebra and to be consistent with the laboratory measurement system.

Only propagation along the magnetic field is considered in detail, but reference is made to the effect of propagation at an angle to the magnetic field. The resonant frequencies (the values of ω for which N becomes infinite) can be found quite simply, and the cut-off frequencies, below which the wave cannot propagate are determined from the values of ω for which N vanishes. For reasons of mathematical convenience N^2 is plotted instead of N, and the plasma frequency ω_p is chosen to be greater than the electron-cyclotron frequency Ω_e. This approximation holds for high densities and low magnetic fields.

The dispersion curves are shown in figure 1.1 for the cold plasma case with propagation along the magnetic field. The dashed curves are the ion and electron modes which only appear for non-zero electron and ion temperature in the parallel propagation case.

When N^2 is less than zero, the waves are evanescant and the

Fig. 1.1 Dispersion curves for an infinite cold plasma.

Parameters are:

$N = \frac{c}{V}$ the refractive index.

ω the wave frequency.

$\omega_1^o = \left(\frac{4\pi ne^2}{m_e}\right)^{\frac{1}{2}}$ the plasma frequency (ω_p).

$\omega_2^o = \left(\omega_p^2 + \frac{1}{4}(\Omega_e+\Omega_i)^2\right)^{\frac{1}{2}} + \frac{1}{2}(\Omega_e-\Omega_i)$

$\omega_3^o = \left(\omega_p^2 + \frac{1}{4}(\Omega_e+\Omega_i)^2\right)^{\frac{1}{2}} - \frac{1}{2}(\Omega_e-\Omega_i)$

$A = \frac{c}{V_A}$ where V_A is the Alfven velocity.

$M = \frac{m_i}{m_e}$

$\Omega_i = \frac{eB}{m_i}$ the ion cyclotron frequency

$\Omega_e = \frac{eB}{m_e}$ the electron cyclotron frequency

$\beta_e^2 = \frac{\gamma_e T_e}{m_e C^2}$

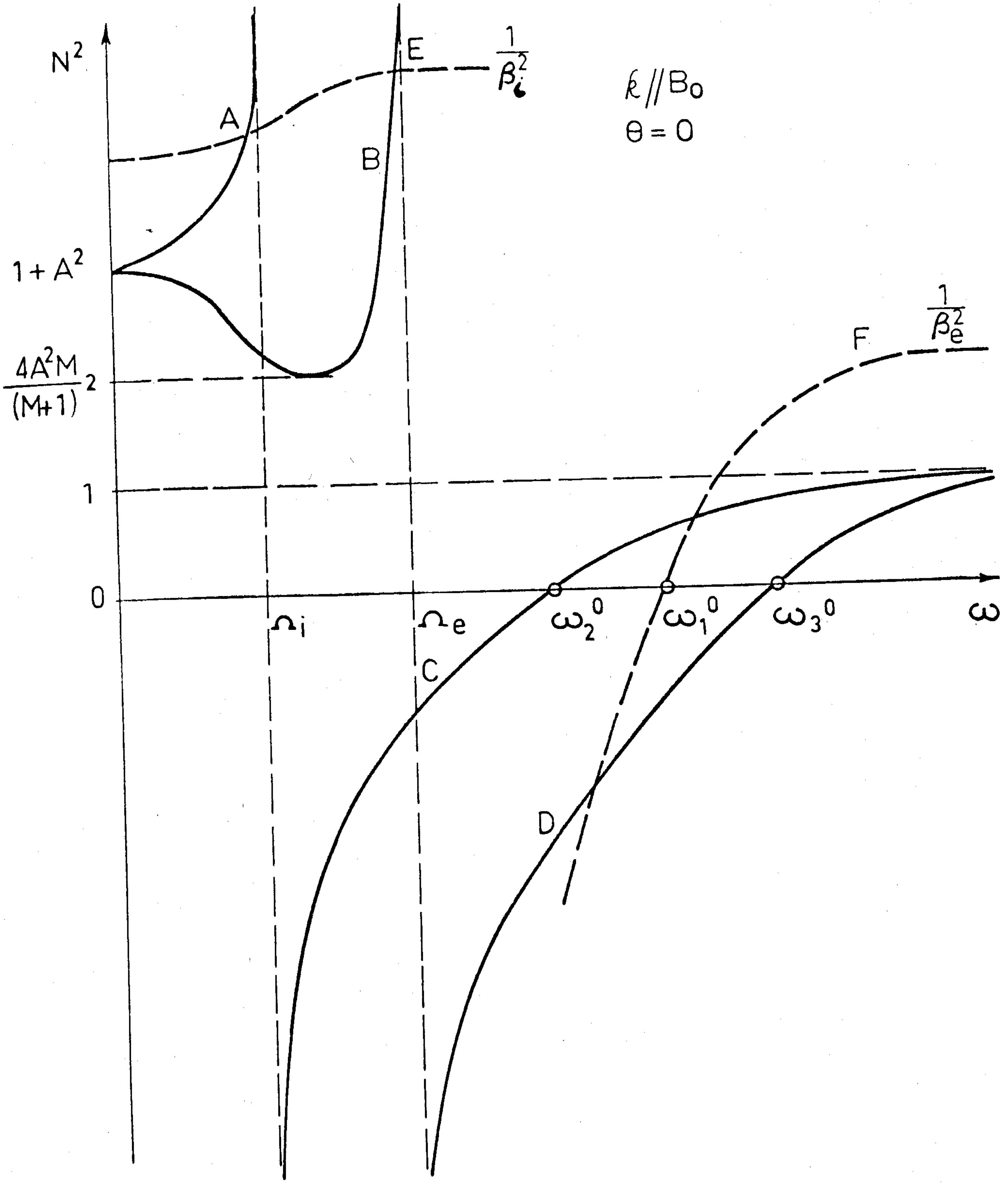
$\beta_i^2 = \frac{\gamma_i T_i}{m_i C^2}$

where γ_e and γ_i are the specific heats of the electrons and ions.

A and C correspond to waves with lefthand polarization.

B and D correspond to waves with righthand polarization.

E and F correspond to the ion and electron waves in a thermal plasma.



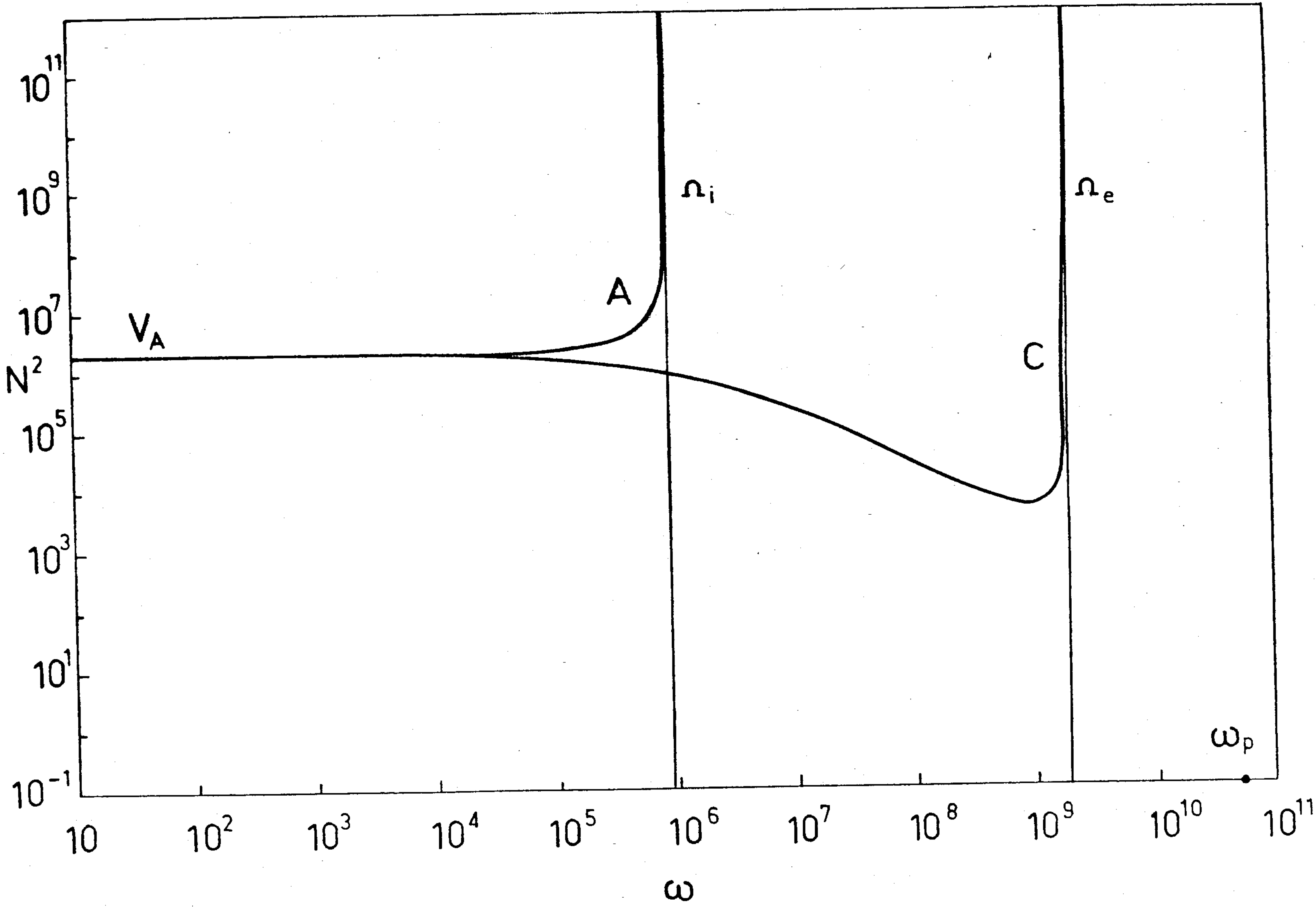
N^2
$\frac{1}{\beta_i^2}$
E
$k // B_0$
$\theta = 0$
A
B
$1 + A^2$
$\frac{1}{\beta_e^2}$
F
$\frac{4A^2M}{(M+1)^2}$
1
0
Ω_i
Ω_e
ω_2^0
ω_1^0
ω_3^0
ω
C
D

In deducing the polarization of the different waves, the convention is adopted in which the direction of the magnetic field is positive ($\underline{k}$ and $\underline{B}_o$ in the same direction). In the cold plasma case there are four roots of the dispersion relation designated A, B, C, and D in figure 1.1. These waves have resonances ($N^2 \to \infty$) at the ion and electron-cyclotron frequencies and cut-offs ($N^2 \to 0$) at the plasma frequency and frequencies related to the plasma frequency. The curves A and C correspond to waves with left-hand polarization, whereas curves B and D correspond to waves with right-hand polarization. In a real plasma (i.e. one where the particle velocity distribution is considered), the dispersion relation becomes of sixth order and two more modes of oscillation appear, the ion and electron waves, designated E and F in figure 1.1. These modes approach the asymptotes $\frac{1}{\beta_i^2}$ and $\frac{1}{\beta_e^2}$ respectively which are proportional to the ion and electron temperatures. The effect of changing the angle θ between the wave vector $\underline{k}$ and the magnetic field $\underline{B}_o$ is clearly shown by Shafranov (fig. 6 in his review (1967)), as is the effect of changing the plasma density. The curves C and D are designated the ordinary and extraordinary modes for $\theta = \pi/2$. For low frequencies ($\omega < \Omega_i$), the modes A and B are called the torsional (or slow), and compressional (or fast) Alfven waves. As $\omega \to 0$ these two modes propagate at approximately the same velocity, the Alfven velocity V_A, and are called, simply, Alfven waves (Alfven (1942)).

In figure 1.2 the dispersion relation for the left and right polarized waves is plotted for a specific case, to show the variation of refractive index with frequency. The gas is Hydrogen with $\frac{m_i}{m_e} = 1836$ in a D.C. magnetic field of 100 gauss. Full ionization is assumed with

Fig. 1.2

Dispersion curves for the left (A) and right (C) polarized waves in a fully ionized infinite hydrogen plasma. Waves propagate along the magnetic field B_o = 100 gauss in a plasma with a density of 10^{12} charged particles/cc.



10^11
10^9
10^7
10^5
10^3
10^1
10^-1
N^2
V_A
A
Ω_i
Ω_e
C
ω_p
10
10^2
10^3
10^4
10^5
10^6
10^7
10^8
10^9
10^10
10^11
ω

no resistivity and an electron and ion density of 10^{12} particles/cc. Thus the parameters V_A, Ω_i, Ω_e, and ω_p are defined giving $\frac{\omega_p}{\Omega_e} = 32$. To allow the interesting parts of the curves to be shown a log-log plot is used. The dispersion relation deduced by Spitzer is used to determine the curves. A slightly different form of this relation is deduced for the right-hand wave in chapter 3.

The curve A has a resonance at the ion-cyclotron frequency Ω_i and at frequencies slightly lower than Ω_i represents the ion cyclotron wave. It has been theoretically investigated by Stix and experimental studies have been carried out in many laboratories, Hooke et al. (1961) in the United States, Jephcott and Stocker (1962) in the U.K., Nazarov (1962) et al. in the U.S.S.R., and Nagao and Sato in Japan. Preliminary investigations on the left wave at frequencies up to 0.3 Ω_i in a shock produced hydrogen plasma are described in chapter 2 of this thesis. As these experiments are carried out in a cylindrical plasma, the dispersion of the waves is considerably modified, the main difference in this low-frequency regime being the appearance of a 'waveguide' cut-off frequency for the right-hand wave, below which the wave cannot propagate. This point has been discussed by many of the authors cited in chapter 2 and the cut-off frequency has been studied both experimentally and theoretically by the author.

The curve C which has a resonance at Ω_e, the electron-cyclotron frequency, is the subject of most of this thesis and it will now be dealt with in greater depth. As the wave propagates over a large frequency range it has acquired a number of different names. At frequencies far below Ω_i it is called the simple Alfven wave; at frequencies near Ω_i, the compressional or fast Alfven wave; at frequencies between Ω_i and

Ω_e where ions are relatively immobile and electron inertia can be neglected, the helicon wave; and at frequencies close to Ω_e, the whistler wave or the electron-cyclotron wave.

In figure 1.3 the dispersion curves are plotted for different approximations to the plane wave dispersion relation, which is deduced in chapter 3. The plasma parameters are the same as those for figure 1.2.

The dispersion equation for these waves as derived in chapter 3, section 1 is:

$$\left(\frac{k}{k_o}\right)^2 \left(1 + \frac{\Omega_i}{\omega} - \frac{\omega}{\Omega_e} + \frac{i}{\Omega_e \tau}\right) = \frac{1}{(1-v^2/c^2)} \qquad (1.1)$$

where $k_o = \dfrac{4\pi ne\omega}{B_o}$

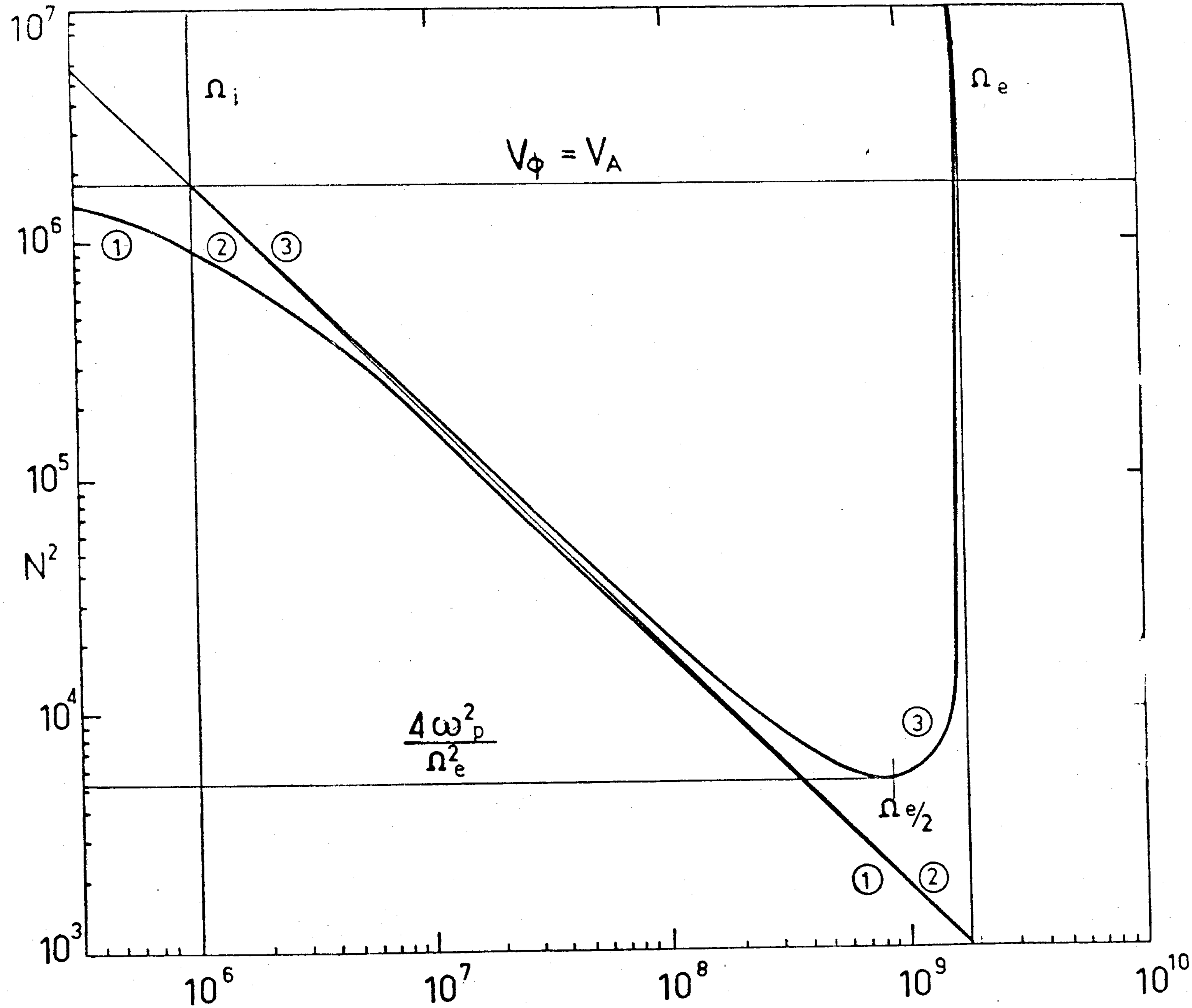
We neglect the displacement current term i.e. the wave velocity << velocity of light, and resistivity is considered negligible.

Introducing the plasma frequency ω_p, equation 1.1 then becomes:

$$\frac{k^2c^2\ \Omega_e}{\omega\ \omega_p^2} \left(1 + \frac{\Omega_i}{\omega} - \frac{\omega}{\Omega_e}\right) = 1. \qquad (1.2)$$

Substituting $\dfrac{k^2c^2}{\omega^2} = N^2$, the square of the refractive index, equation (1.2) yields:

$$N^2 \left(1 + \frac{\Omega_i}{\omega} - \frac{\omega}{\Omega_e}\right) = \frac{\omega_p^2}{\Omega_e \omega} \qquad (1.3)$$



10^7
10^6
10^5
10^4
10^3
N^2
Ω_i
Ω_e
$V_\phi = V_A$
$\frac{4\omega_p^2}{\Omega_e^2}$
$\Omega_e/2$
①
②
③
10^6
10^7
10^8
10^9
10^{10}

Fig. 1.3

Different approximations to the dispersion of the right polarized wave. The same conditions prevail as for figure 1.2.

Curve 1 : Compressional Alfven wave.

Curve 2 : Helicon wave.

Curve 3 : Electron cyclotron wave.

The three regions of different approximation to equation (1.1), depending on the relative magnitude of the frequencies Ω_i, Ω_e, and ω_p, are set out below.

1.2 Compressional Alfven Waves

In this regime, the electron inertia is neglected and the term $\frac{\omega}{\Omega_e}$ is dropped out of equation 1.3. The equation describing the dispersion of these waves is:

$$N^2 = \frac{\omega_p^2}{\Omega_e[\Omega_i+\omega]} \qquad (1.4)$$

and when $\omega << \Omega_i$

$$N^2 = \frac{\omega_p^2}{\Omega_e\Omega_i} ,$$

and the phase velocity of the wave becomes:

$$V_\phi = \frac{c}{N}$$

$$= \frac{c}{\omega_p} \sqrt{\Omega_e\Omega_i}$$

$$= V_A$$

the Alfven velocity. These waves have been investigated by Spillman (1963), Cross (1968) and others. The compressional wave can be seen to arise from a combination of the $\underline{v} \times \underline{B}$ and $\underline{j} \times \underline{B}$ terms of the Ohms law equation

(Spitzer), which reduces, in this case, to

$$\underline{E} = \frac{1}{ne}\,\underline{j} \times \underline{B} - \underline{v} \times \underline{B}$$

These waves are labelled (1) in figure 1.3. The low-frequency Alfven wave comes from the simplest form of Ohms law

$$\underline{E} + \underline{v} \times \underline{B} = 0.$$

1.3 Helicon Waves

For these waves the ions are considered immobile and electron inertia is neglected ($\Omega_i << \omega << \Omega_e$) and the terms $\frac{\Omega_i}{\omega}$ and $\frac{\omega}{\Omega_e}$ are dropped from equation 1.3. The resulting dispersion relation then acquires the elegantly simple form:

$$N^2 = \frac{\omega_p^2}{\Omega_e \omega} \qquad (1.5)$$

These waves are labelled (2) in figure 1.3, from which the approximation can be seen to be quite accurate in the central region between Ω_i and Ω_e. If the right-hand side of equation 1.5 is expanded in terms of the plasma parameters

$$N^2 = \left(\frac{c}{V_\phi}\right)^2 = \frac{4\pi ne}{\omega B}\,, \qquad (1.6)$$

and it can be seen that the velocity is independent of the electron or ion mass. The lines of force are helical and rotate carrying the electrons with them, hence the name 'helicon waves' proposed by Aigrain (1960).

Referring back to Ohms law, these waves are described by the approximate form:

$$E = \frac{1}{ne} \underline{j} \times \underline{B}_o .$$

1.4 Electron-Cyclotron Wave

This wave propagates at frequencies below, and up to, the electron-cyclotron frequency. The ion inertia is neglected and the term $\frac{\Omega_i}{\omega}$ disappears from equation 1.3. In ionospheric physics, since the magnetic field is quite low, these waves propagate at frequencies in the audio region and in general are heard on receivers as a falling tone. The term 'whistlers' has been applied to these waves and they have been investigated by Storey (1952) and Ratcliffe (1962). In laboratory plasmas, these waves are generally propagated as microwaves or high-radio frequency waves (Dellis and Weaver (1962)).

The dispersion relation is:

$$N^2 = \frac{\omega_p^2}{\omega\Omega_e[1-\frac{\omega}{\Omega_e}]}$$

and also describes the propagation of waves in the helicon regime. This is a direct result of the inclusion of the $j \times B_o$ term in Ohms law, which in this frequency region becomes:

$$\underline{E} = \frac{1}{ne} \underline{j} \times \underline{B}_o + \frac{m_e}{ne^2} \frac{\partial \underline{j}}{\partial t}$$

The only term in the Ohms law which has not been discussed is the term through which the electron inertia manifests itself. If the Ohms law is written for this case

$$E = \frac{m_e}{ne^2} \frac{\partial j}{\partial t}$$

and an equation for the perturbing field $\underline{b}$ is formed, it can be seen that this term only describes oscillations at the plasma frequency. However, when the Hall current term $\underline{j} \times \underline{B}$ is linked with the electron inertial term, the electron cyclotron resonance appears. As can be seen from figure 1.3, the electron inertial term has a large effect on the dispersion relation for quite a wide frequency range (curve 3).

There is a minimum in the refractive index at a frequency $\omega = 0.5\ \Omega_e$. The effect of this velocity maximum in the case of cylindrical geometry is discussed in chapter 3, section 4.

1.5 Discussion

This rather long introduction has been included to show the different regions of validity for the approximations applied to the dispersion relation for the right-hand polarized wave. This thesis is mainly concerned with the helicon regime, but the importance of the electron inertial term is shown theoretically in chapter 3 and in chapter 4 where some results of Klosenberg, McNamara, and Thonemann (1965) are investigated.

Although quite long literature surveys have been given elsewhere (Christiansen (1969)), a brief survey of helicon and whistler wave propagation is given below.

The name helicon was suggested by Aigrain (1960) to describe an electromagnetic wave which propagates in solids at low temperatures at frequencies $\omega \ll \Omega_e$. These waves were initially investigated as resonances or standing waves in sodium at liquid helium temperatures by Rose, Taylor, and Bowers (1962). Until 1964 the plane wave dispersion relation, modified by geometrical factors, was used to explain the experimental results. In 1964/65 Legendy, in the U.S.A., and Klozenberg, McNamara, and Thonemann (sometimes referred to as K.M.T. in this thesis) in the U.K., independently presented theories for the propagation of helicons in a cylindrical magnetoplasma. Both papers present effectively the same theory and spectulate on the zero resistivity case with vacuum boundaries, but K.M.T. also present calculated dispersion and attenuation curves.

Helicon wave propagation in indium has been described by Harding and Thonemann (1965) while Facey and Harding (1966) have investigated standing helicon waves in a cylinder of indium.
The predictions of the K.M.T. theory were in good agreement with the results of these two experiments and those of Lehane and Thonemann (1965) who propagated helicon waves in a cylindrical R.F. maintained plasma.

The resonant heating of a plasma by a high-frequency field at the low frequency end of the helicon regime has been described by Chechkin et al. (1965) and more fully investigated by Vasil'ev et al. (1968). The anomolously high damping encountered was originally thought to be due to Cherenkov damping (Shafranov (1958)), but later measurements showed that the linear theory for Cherenkov damping could not be applied. Longitudinal oscillations which propagate almost perpendicularly to the magnetic field which are excited by the helicon and consequently damp the wave, were suggested as possible damping mechanism. A collisional mechanism involving ohmic dissipation of currents flowing at the plasma surface suggested by Klozenberg and Lehane (1965) was considered unlikely as the electrons gain significant energy in a time less than τ, the electron-neutral collision period (Vasilev et. al. 1968).

The above mentioned helicons involved propagation within rigid non-conducting boundaries. A considerable amount of work has been done in plasma cylinders with metal conducting walls. Blevin and Christiansen (1966) showed the effect of radial density gradients in a plasma with conducting walls. Resistive effects were included in the dispersion relation by Ferrari and Klosenberg (1965) for a uniform cylindrical plasma with conducting walls. Davies (1970) and Boswell (1972) have discussed the effect of electron inertia on the dispersion of helicon waves, following the work of Blevin et. al. (1968).

Helicon waves are studied for a number of reasons, apart from the fact that they can be used to measure Hall coefficients in semi-conductors and the explanation of phenomena in the ionosphere. Collisionless phenomena such as Landau and Cherenkov damping can be studied in gaseous plasmas using helicon waves. The anomalous skin effect (Blevin, Reynolds, and Thonemann (1968)) could possibly have a considerable effect on the dispersion of helicons. This possibility has been treated theoretically by Kondratenko (1966), by including thermal effects in the dispersion of the waves.

In the hydromagnetic approximation, the wave fields are considered to be small perturbations in the plasma and do not change the properties of the plasma. However, when thermal effects are included in the theory, the wave fields can accelerate or retard the charged particles of the plasma, thereby changing its density and temperature. Although the difference between simple propagation of waves through a plasma and the effect that the waves have on the plasma parameters is rather artificial, it is convenient for reasons of mathematical simplicity to consider these problems separately.

The problem of forming and heating a plasma is considerable, but since a plasma in a magnetic field can sustain certain types of oscillations, a common method is to transfer energy from an external circuit to some form of allowable oscillation in the plasma. The collisional or collisionless damping of this oscillation then increases the energy of the particles comprising the plasma.

The ion cyclotron wave is generated for this purpose and is propagated into a region where the magnetic field is decreasing. The ion cyclotron frequency decreases proportionally with the magnetic field and the wave suffers collisionless cyclotron damping. The directed energy of the particles is randomized into thermal energy by collisions or by 'phase mixing', discussed by Berger et al. This form of heating involves only the ions, the electrons acquiring energy only through ion-electron collisions. The second chapter of this thesis describes preliminary experiments on the left-hand polarized wave in a shock-produced cylindrical plasma. It is shown that to overcome the experimental difficulties associated with trying to propagate waves at frequencies up to the ion cyclotron frequency, the plasma density must be decreased by about two orders of magnitude.

One way of doing this is excite waves which are right-hand polarized in the helicon regime. The ions can then be considered immobile and the wave energy transferred to the electrons by collisional, Landau or Cherenkov damping mechanisms. A rotating magnetic field has been used by Blevin and Thonemann (1961) to transfer energy to the electrons which rotate, producing an azimuthal current. The axial field produced by this current is opposite in direction to the applied magnetic field and acts as a confining mechanism. A high-frequency oscillation seen on magnetic probe signals was thought to be a standing helicon produced by the applied fields. The densities and temperatures produced by this method of excitation were in the appropriate region required for ion cyclotron wave studies.

It was therefore decided to use a simple transverse high-frequency field, similar to that described by Blevin and Thonemann, to excite a standing helicon wave. In the density range 10^{12} - 10^{13} electrons/cc, the ionization measured was about 2%, being quite high for the modest amount of power applied (600 watts). To the best of the author's knowledge, there have been no previous experimental investigations on standing helicon waves in a gaseous plasma. This form of wave has, however, received considerable attention in solid state physics (Kaner and Skobov (1968)).

Chapter 3 presents a theoretical analysis of helicon wave propagation in an infinite medium and in a uniform cylindrical plasma. The heating of the plasma (ηj^2) by the current of the helicon is shown to change as the frequency increases, allowing an explanation of some of the experimental results presented in chapter 7.

It is further shown that the inclusion of electron inertia, which fixes a maximum velocity in the plane wave case, has a significant effect on the radial structure of the wave when boundaries are imposed on the plasma. The final section in chapter 3 is a theoretical description of standing helicon waves in a non-uniform resistive plasma, including the effects of electron inertia.

A discussion of the dispersion and attenuation of helicon waves bounded by a rigid non-conductor for the case of low resistivity is given in chapter 4.

The vacuum system, magnetic field coils, and the R.F. excitation system are described in chapter 5.

The diagnostic procedures and theory are presented separately in chapter 6.

Chapter 7, the final chapter, includes the results of experimental measurements of the standing helicon waves. The validity of the diagnostic methods and of the linear theory for standing helicons is discussed and compared with the experimental measurements.